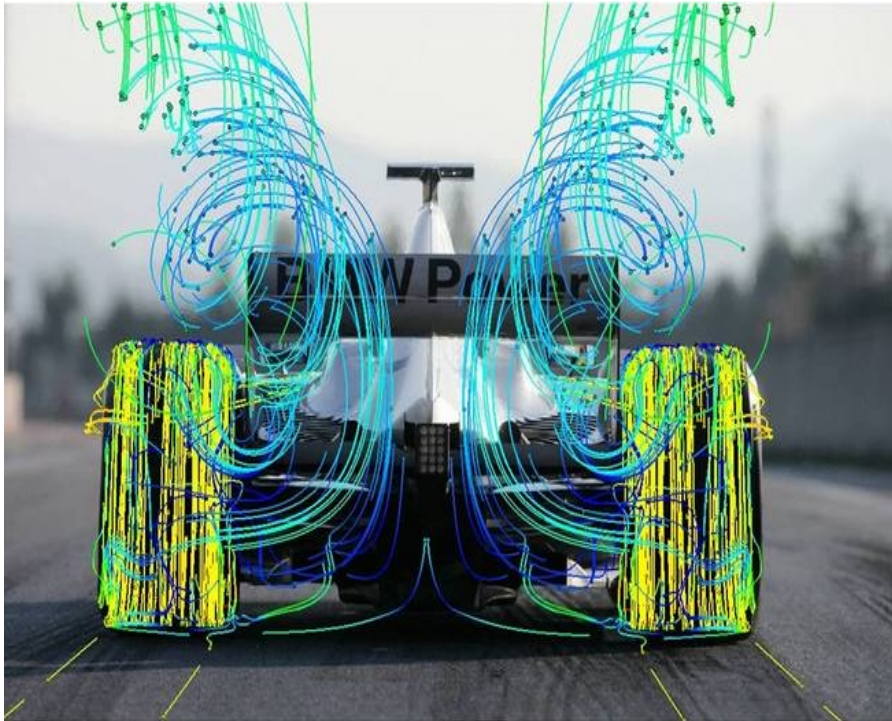


Mystérieuse Turbulence

Bérengère Dubrulle

SPEC

CNRS UMR 3680

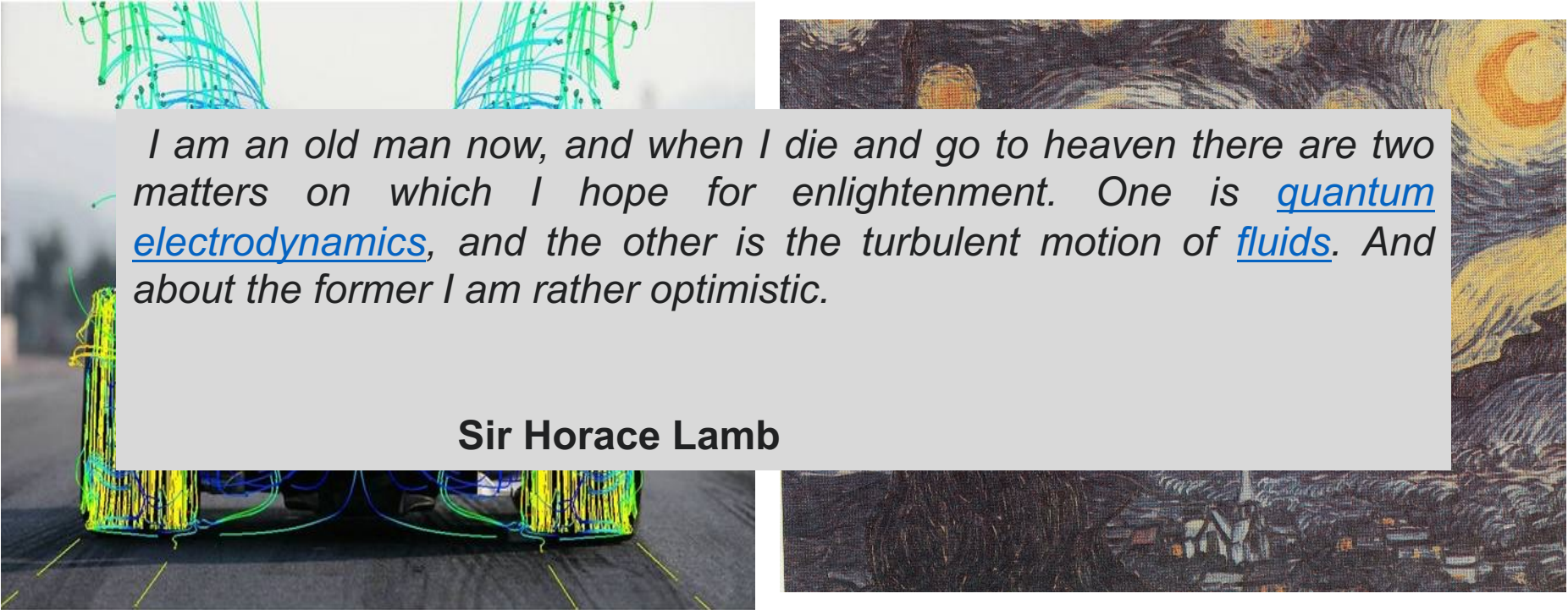


Mystérieuse Turbulence

Bérengère Dubrulle

SPEC

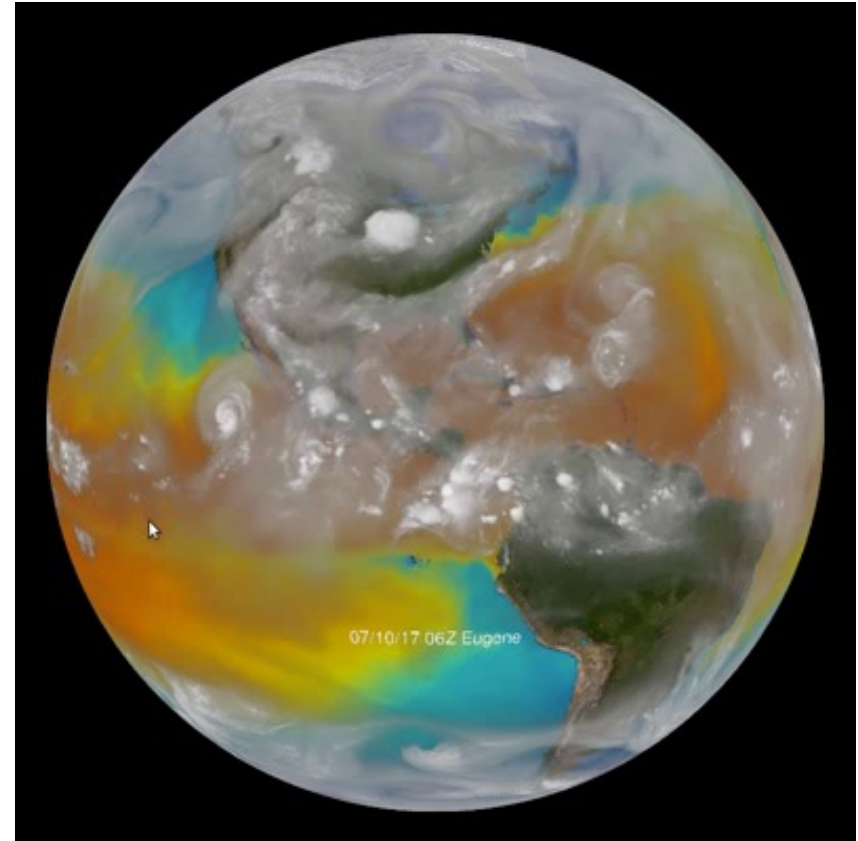
CNRS UMR 3680



I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is [quantum electrodynamics](#), and the other is the turbulent motion of [fluids](#). And about the former I am rather optimistic.

Sir Horace Lamb

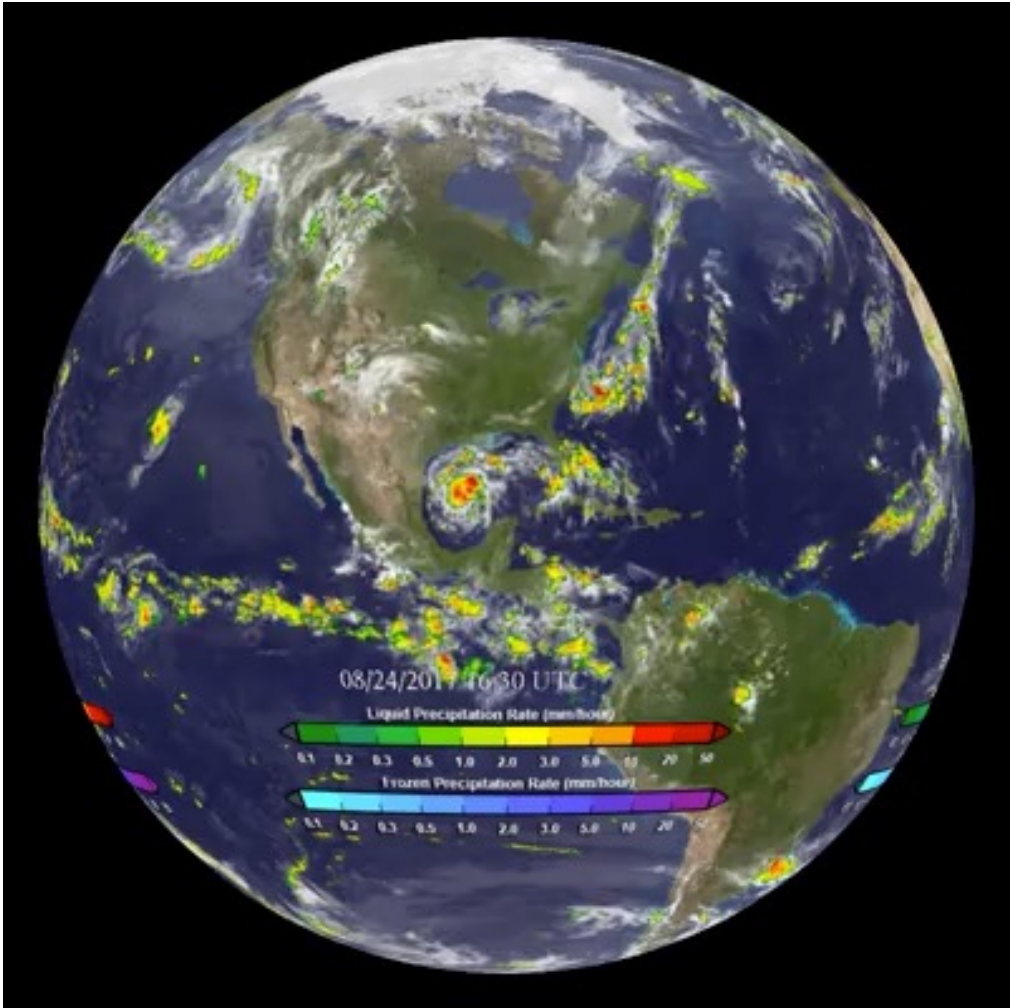
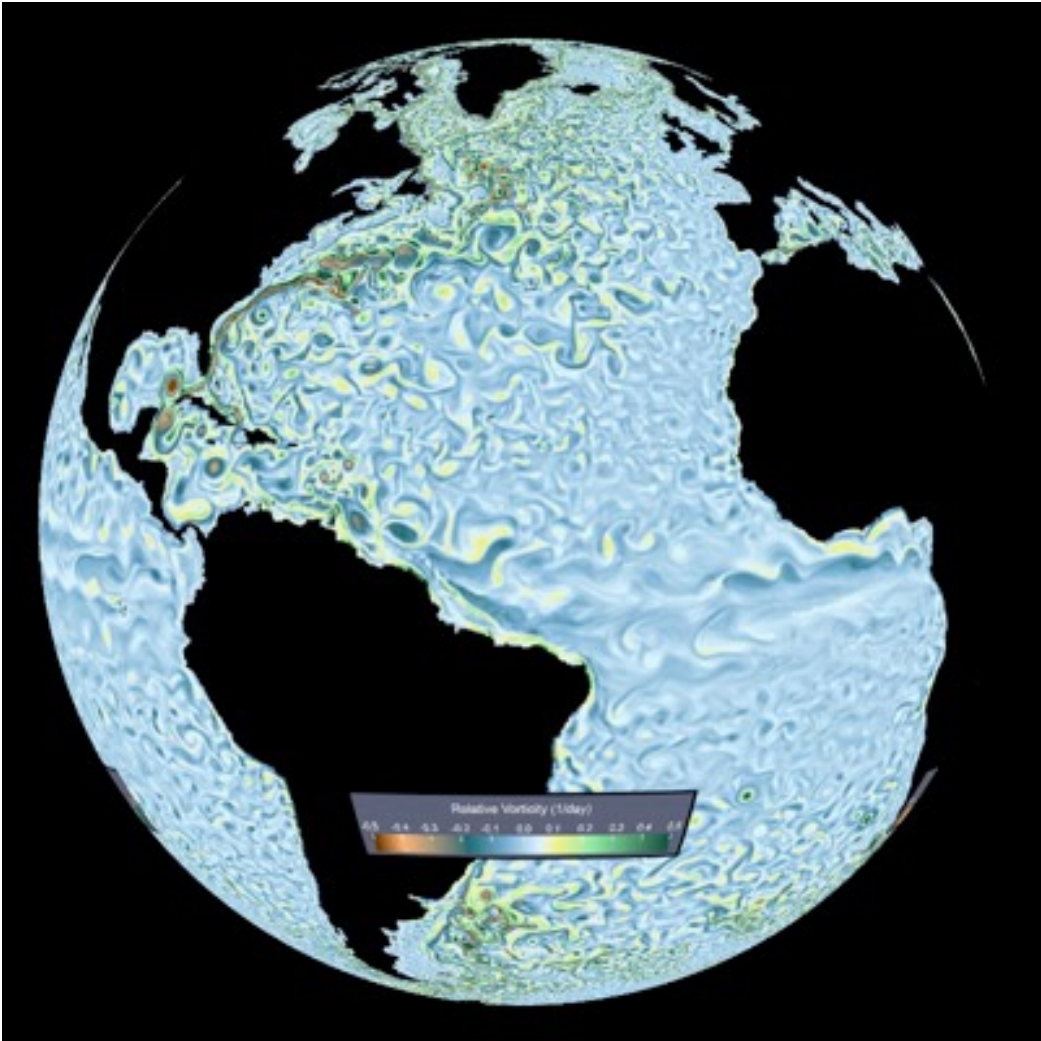
Turbulence....



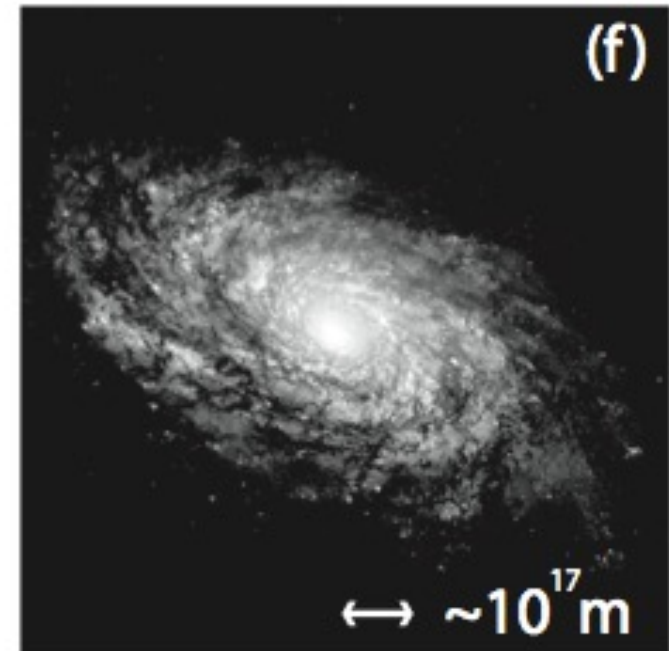
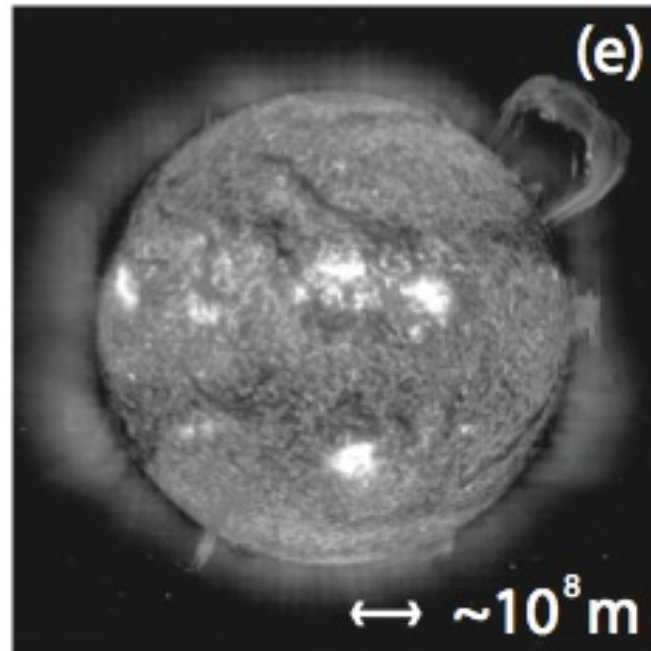
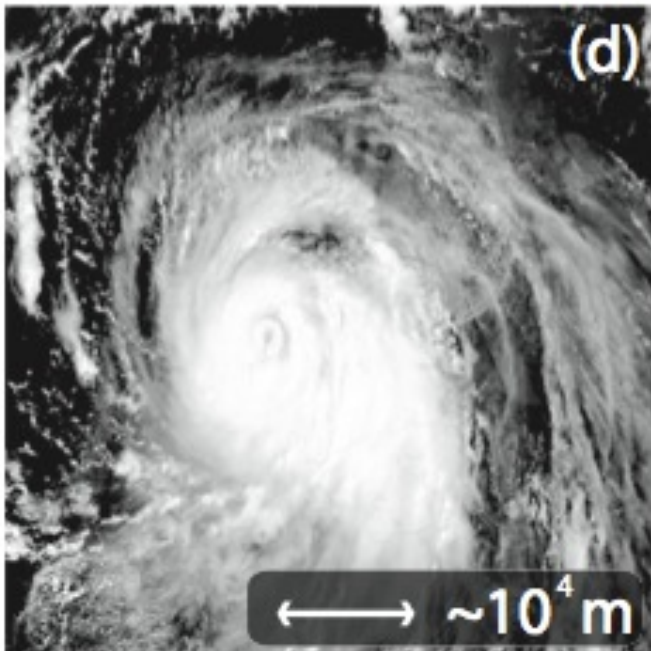
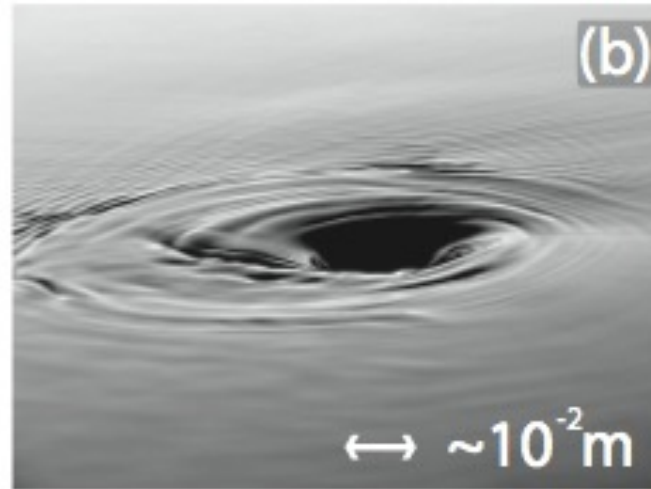
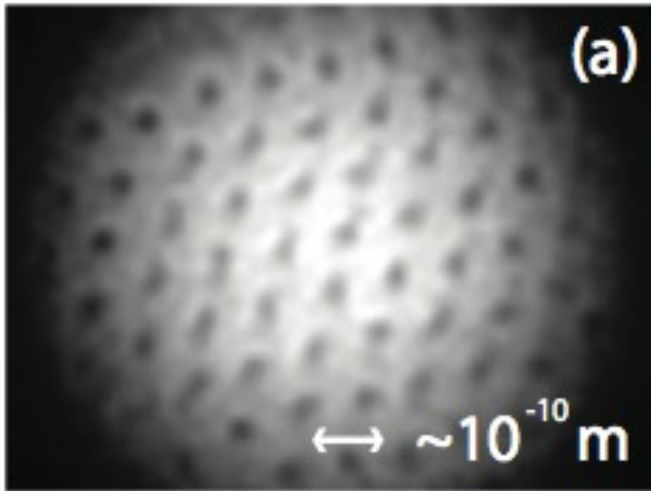
*Définition: La **turbulence** désigne l'état d'un fluide, liquide ou gaz dans laquelle la vitesse présente un caractère désordonné et tourbillonnaire.*



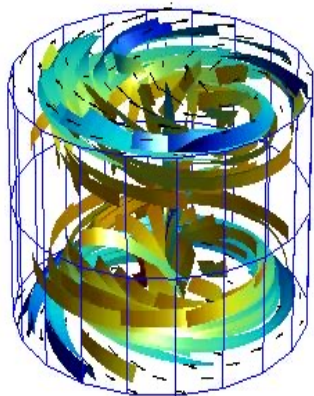
Tourbillons sur la Terre...



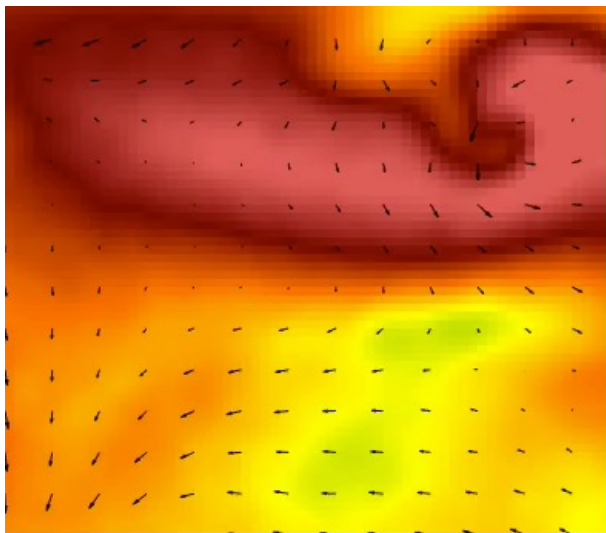
Tourbillons dans l'Univers



Tourbillons dans le laboratoire



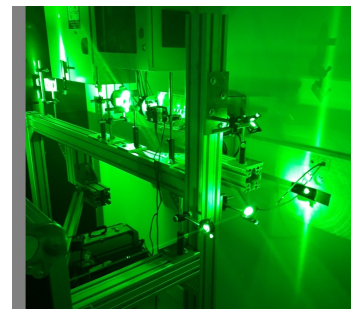
Mesures SPIV



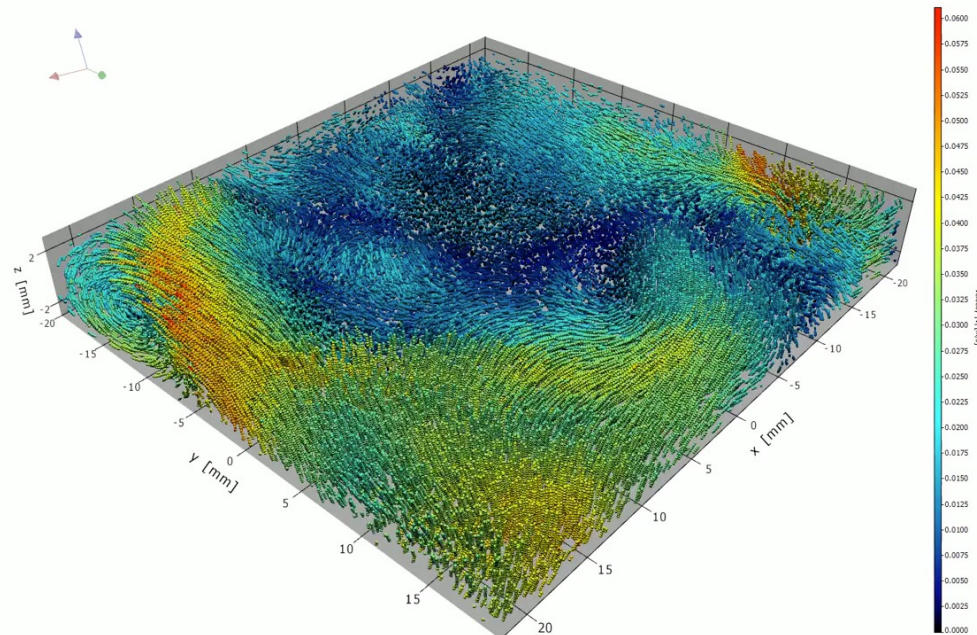
4 Cameras rapides



Laser



Mesure 4D-PTV

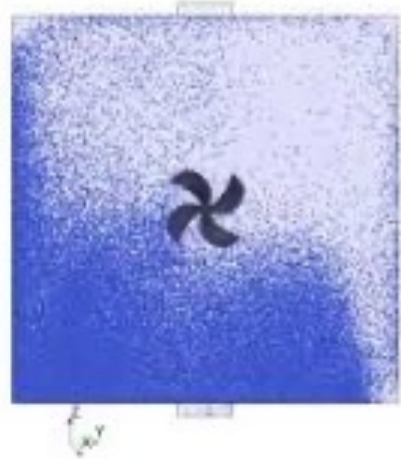


Le Mauvais côté de la turbulence

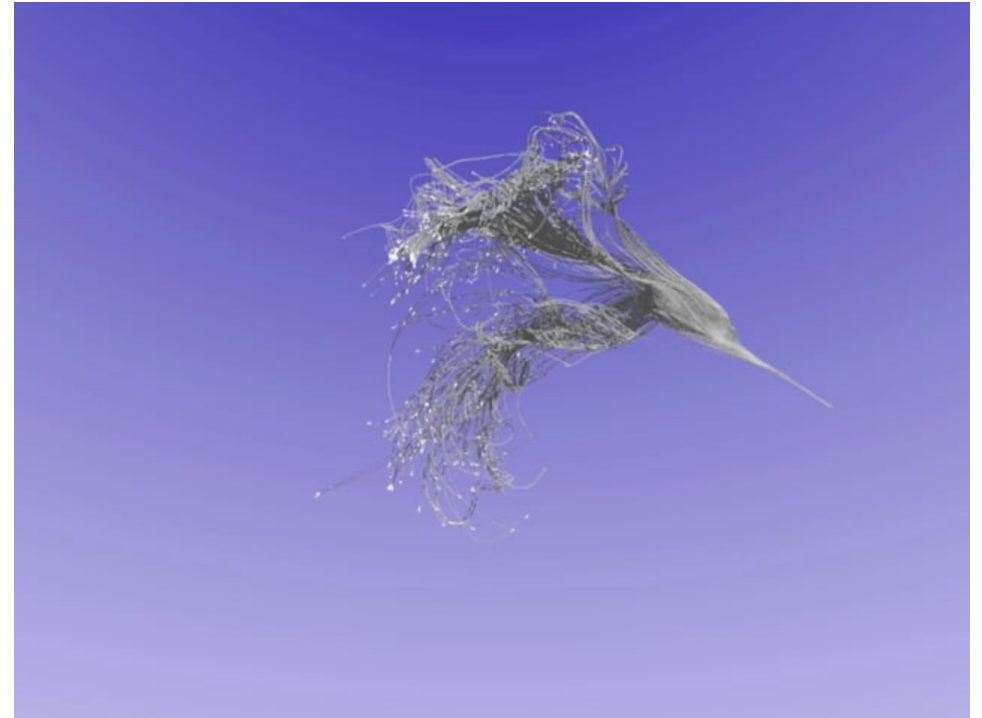
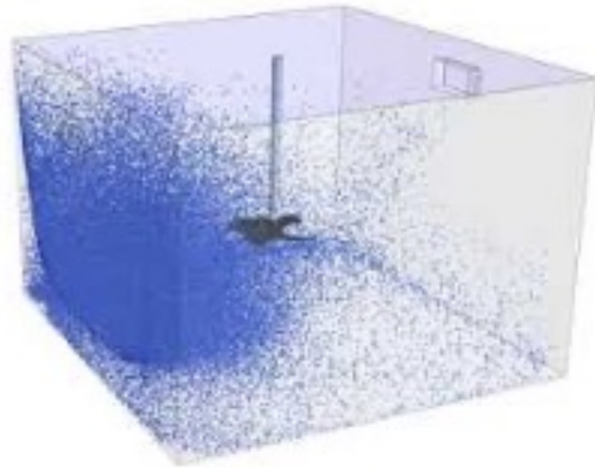


Le bon coté de la turbulence: Dispersion de particules

THINK Fluid Dynamix



With turbulent dispersion model
Solution Time 30.98 (s)



$$\langle (\Delta x)^2 \rangle (t) \sim \langle (\Delta x)^2 \rangle (0) e^{\lambda t}$$

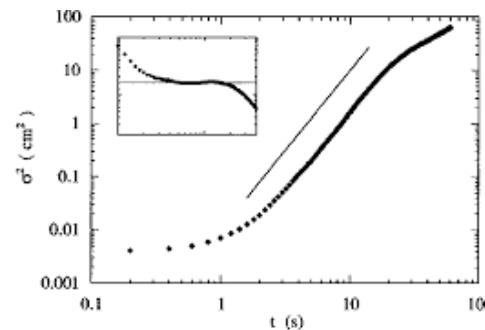
Mémoire de la dispersion initiale

Dispersion chaotique

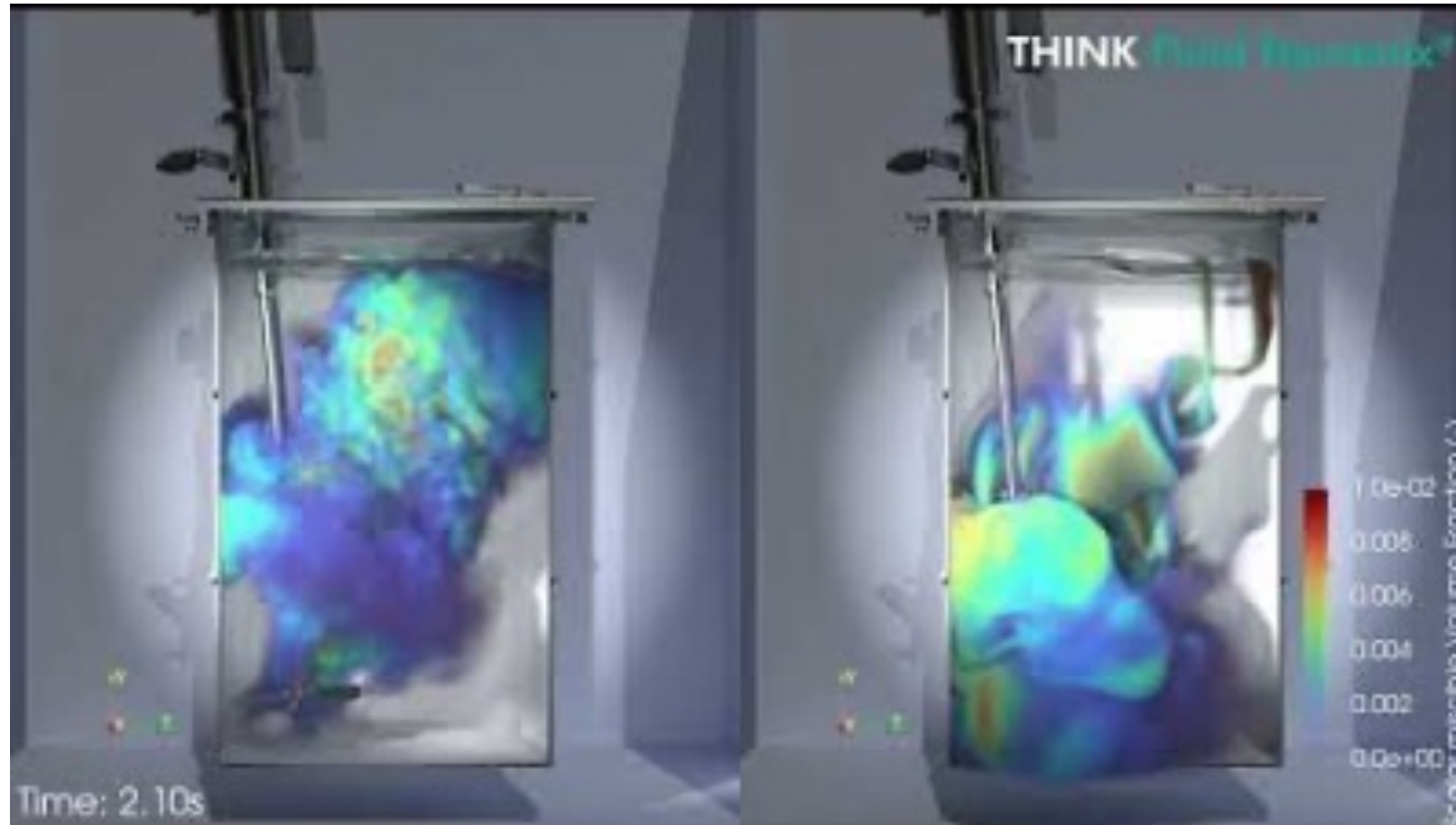
$$\langle (\Delta x)^2 \rangle (t) \sim \epsilon t^3$$

Indépendant de la dispersion initiale

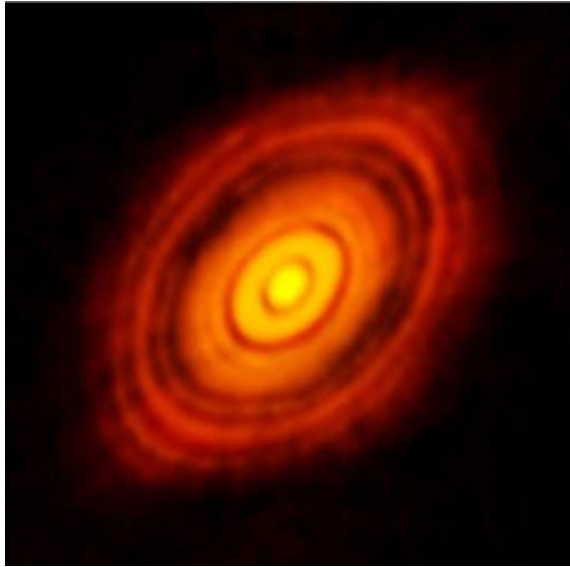
Dispersion turbulente



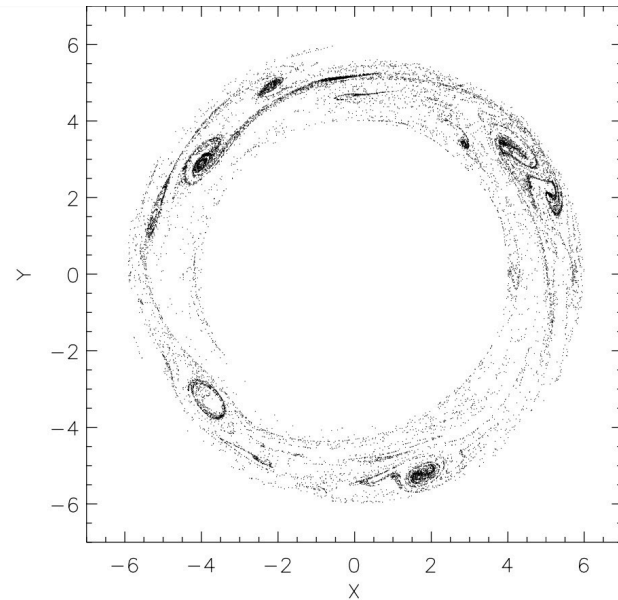
Le bon côté de la turbulence: Mélange turbulent vs laminaire



Turbulence et formation du système solaire

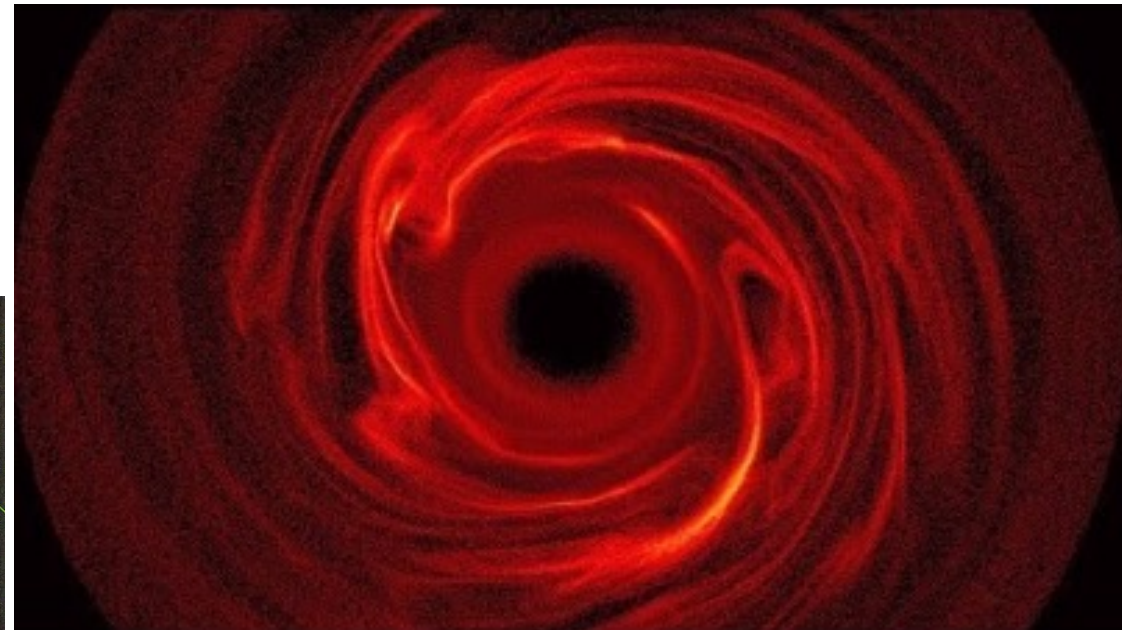
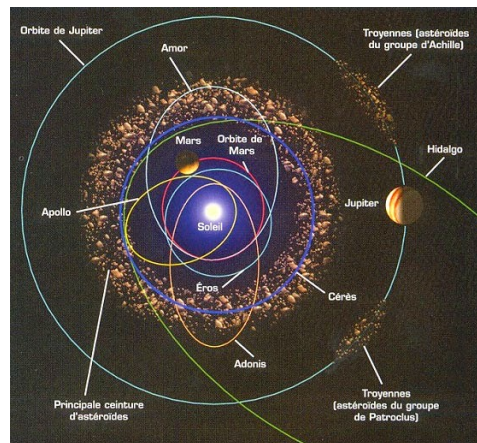


Credit: Alma



Credit: Bracco et al, 1998

Importance de la turbulence pour former les planètes
Importance de la turbulence pour mélanger les éléments



Equations de Navier-Stokes

*Développées il y a 200 ans par Navier (Ecole des Ponts) et Stokes
À la suite des travaux de Euler*

$$\vec{\nabla} \cdot \vec{u} = 0$$

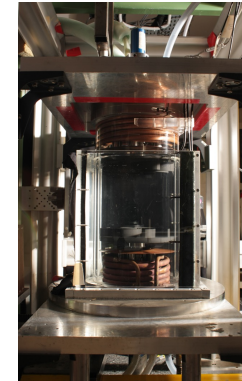
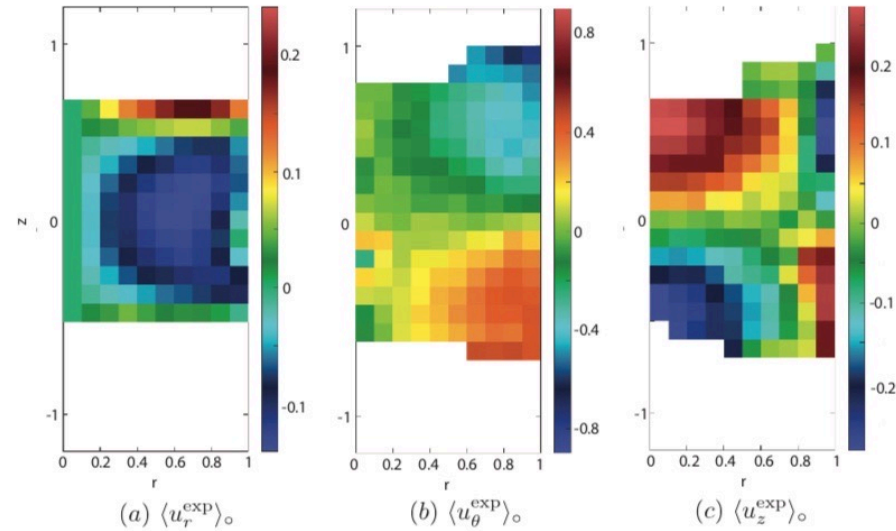
$$\partial_t \vec{u} + (\vec{u} \cdot \vec{\nabla}) \vec{u} = -\frac{1}{\rho} \vec{\nabla} p + \nu \Delta \vec{u}$$

$$\text{Re} = \frac{(u \nabla u)}{\nu \Delta u} = \frac{LU}{\nu}$$

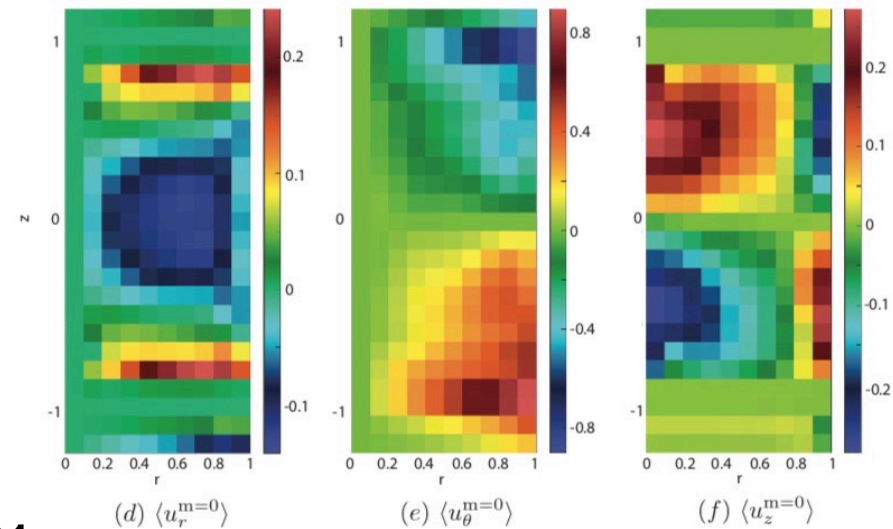
Turbulence: $\text{Re} \gg 1$

Modèle vs Mesures: vitesse moyenne à $Re=10^3$

Lab



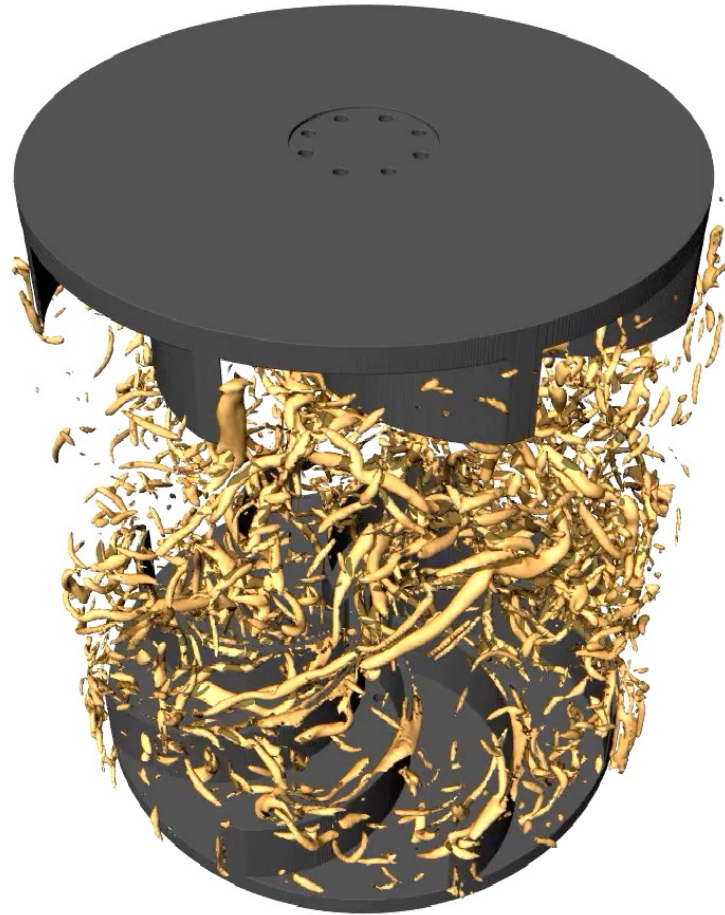
DNS



$$\begin{aligned} \vec{\nabla} \cdot \vec{u} &= 0 \\ \partial_t \vec{u} + (\vec{u} \cdot \vec{\nabla}) \vec{u} &= -\frac{1}{\rho} \vec{\nabla} p + \nu \Delta \vec{u} \end{aligned}$$



Simulations numériques : tourbillons à $Re=10^3$



Avec les simulations, nous
Pouvons voir des choses difficilement
accessibles
aux expériences....

Courtesy H. Faller and A. Harikrishnan

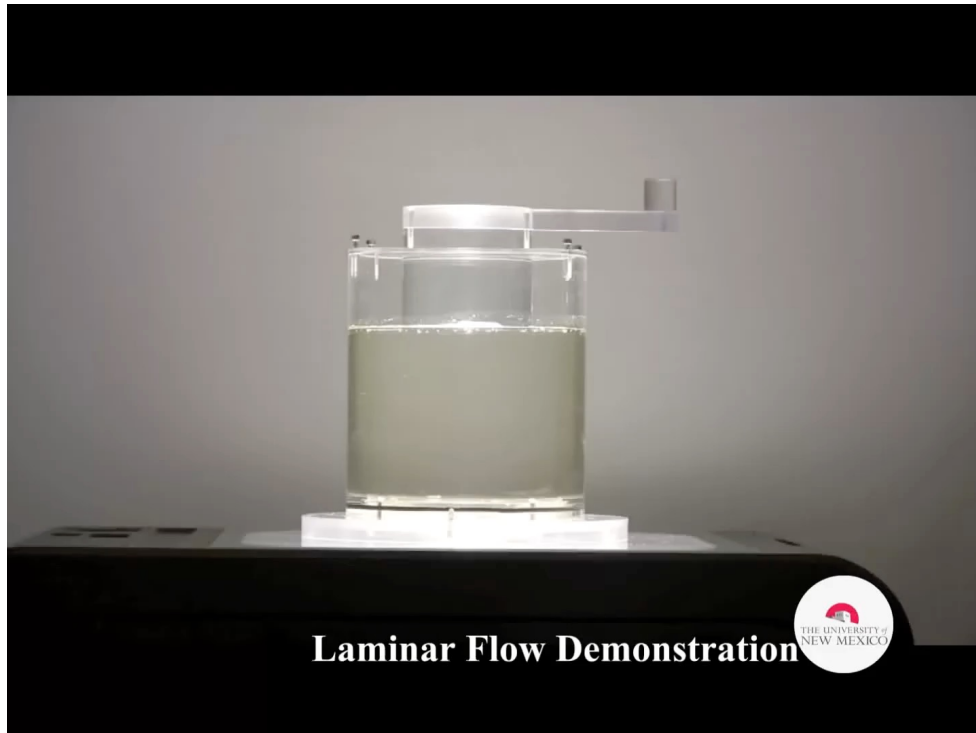
On a les equations... tout est résolu?



Mystère : réversibilité

Écoulement laminaire

Reversible



Écoulement turbulent

Irreversible

D'où vient l'irréversibilité de la turbulence?

Quel est le problème avec la turbulence?



Navier-Stokes et Problème mathématique

The Millennium Problems

The Seven
Greatest
Unsolved
Mathematical
Puzzles
Time



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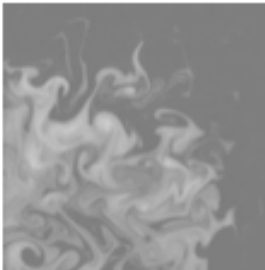
Navier-Stokes Equation

Waves follow our boat as we meander across the lake, and turbulent air currents follow our flight in a modern jet. Mathematicians and physicists believe that an explanation for and the prediction of both the breeze and the turbulence can be found through an understanding of solutions to the Navier-Stokes equations. Although these equations were written down in the 19th Century, our understanding of them remains minimal. The challenge is to make substantial progress toward a mathematical theory which will unlock the secrets hidden in the Navier-Stokes equations.

[The Millennium Problems](#)

[Official Problem Description — Charles Fefferman](#)

[Lecture by Luis Caffarelli \(video\)](#)



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Theorie:

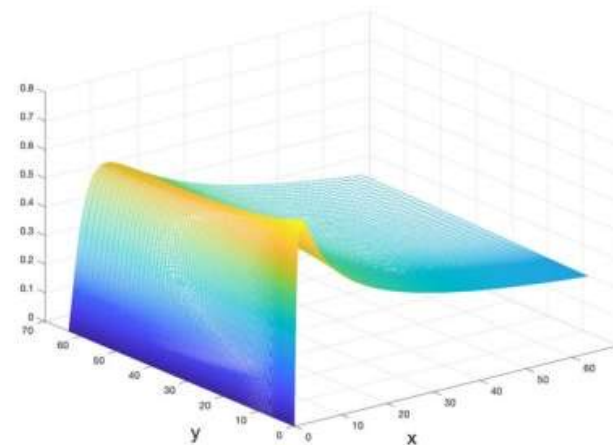
Les equations De Navier-Stokes sont-elles bien posées?
(y a t il des singularites?)

Exemple de formation de singularité

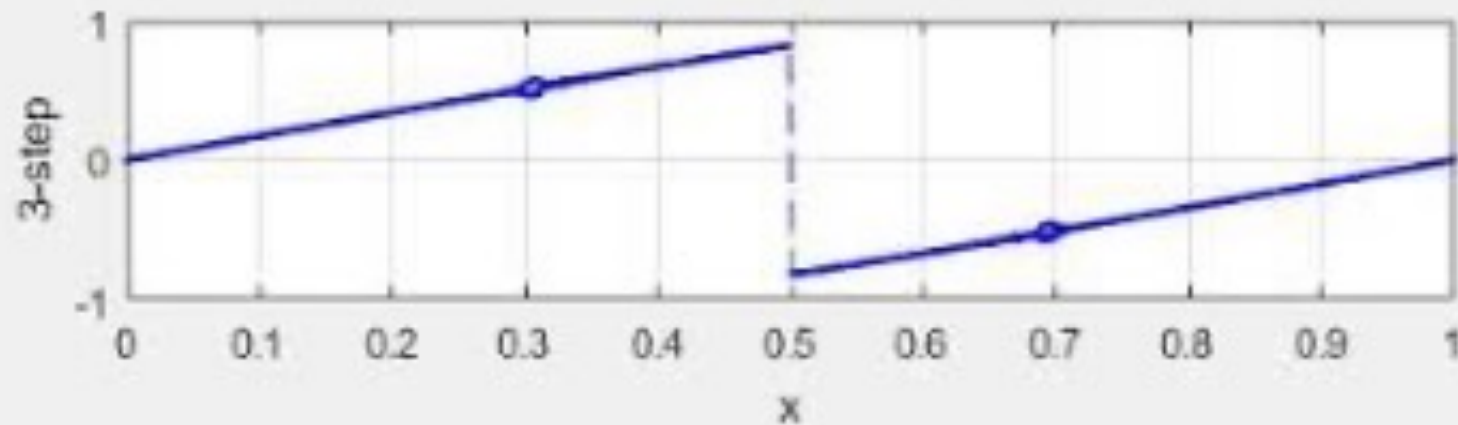
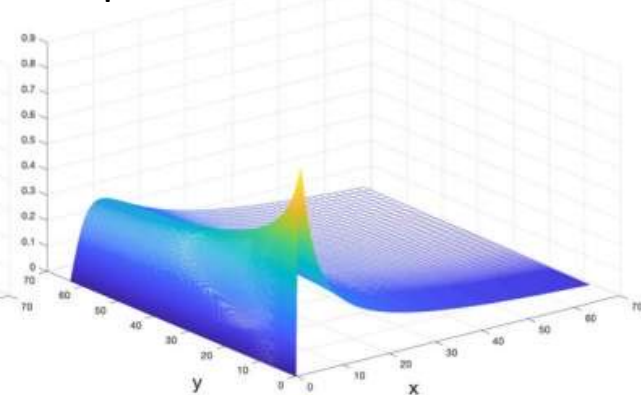
Equation de Burgers 1D

$$\text{Burgers: } \partial_t u + u \partial_x u = \nu \Delta u$$

Discontinuité de la vitesse



Equation d'Euler 3D

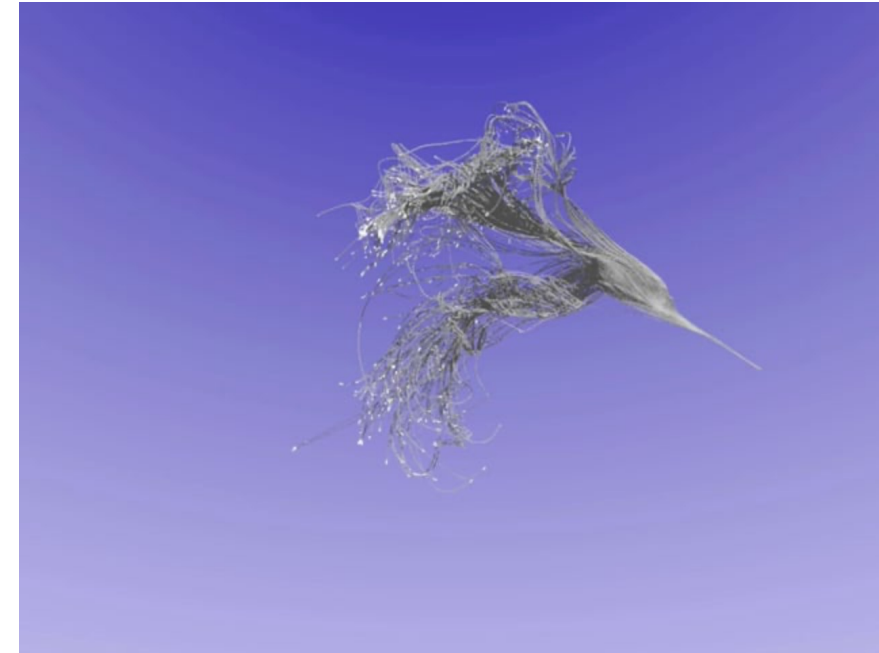
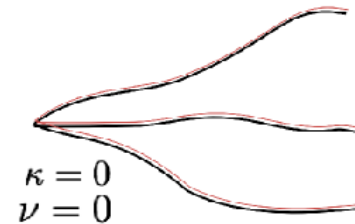
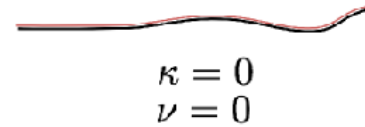
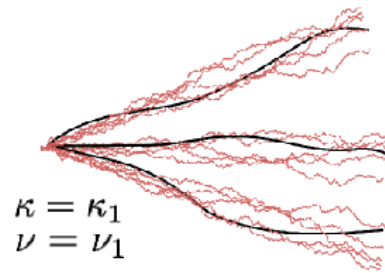
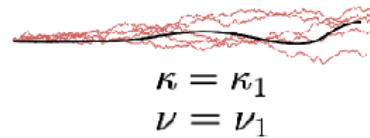
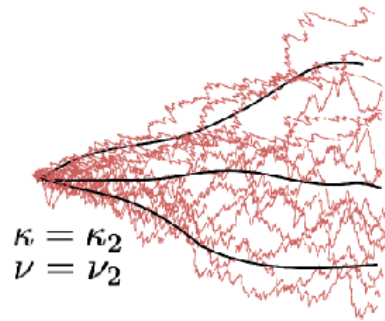
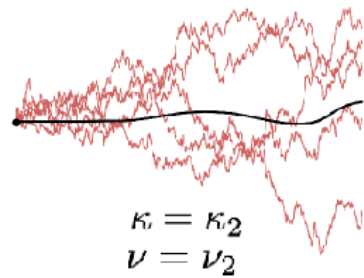


$$\text{Euler: } \partial_t u + u \partial_x u = -\partial_x p$$

Divergence de la dérivée de la vorticité

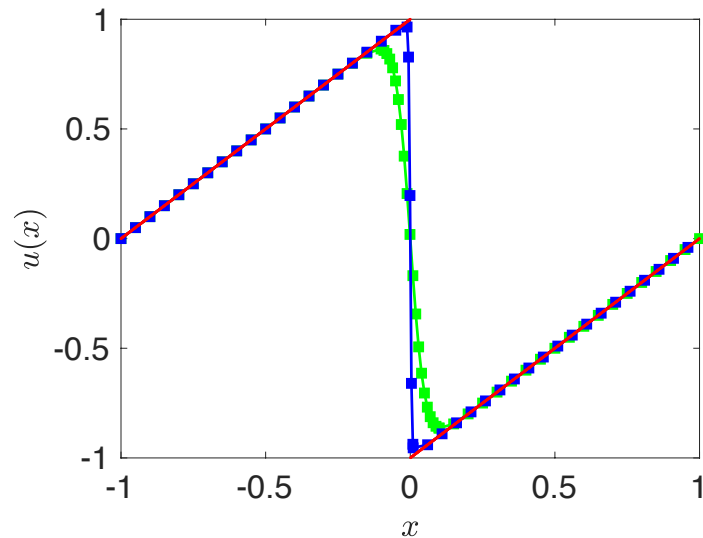
Problèmes induits par les singularités

- Elles peuvent briser l'unicité de la solution et induire de la stochasticité spontanée

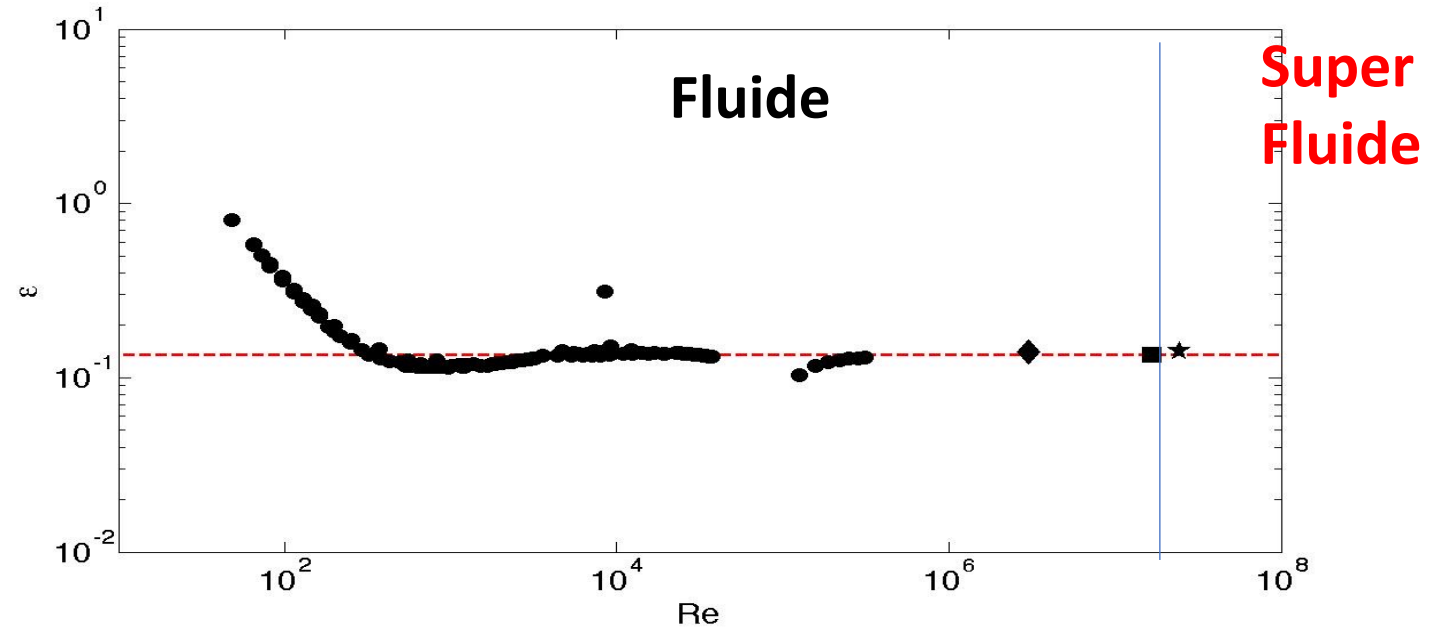


Problèmes induits par les singularités

- Elles provoquent une dissipation et induisent une irréversibilité spontanée

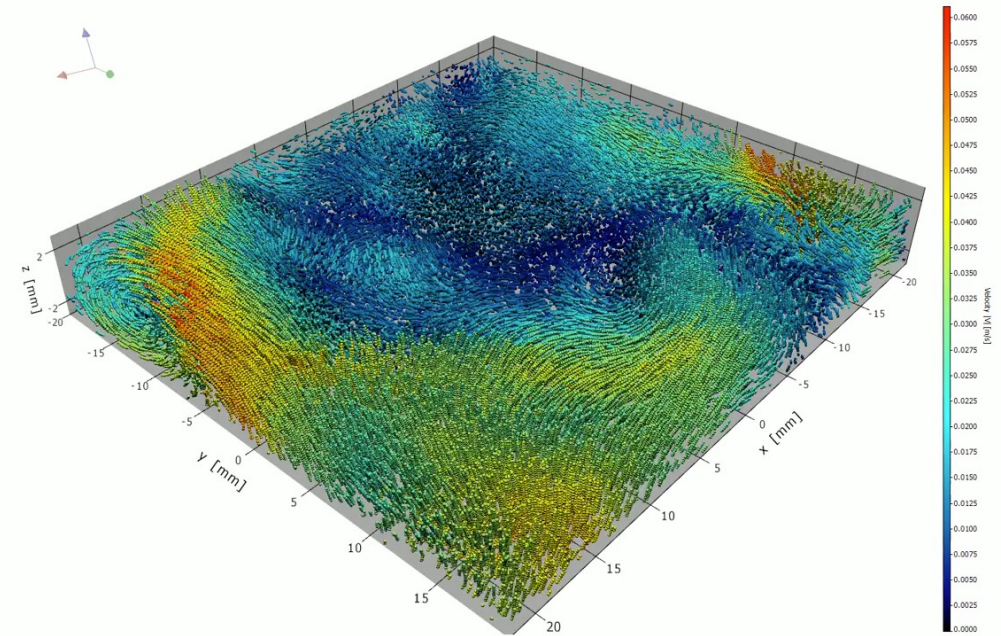
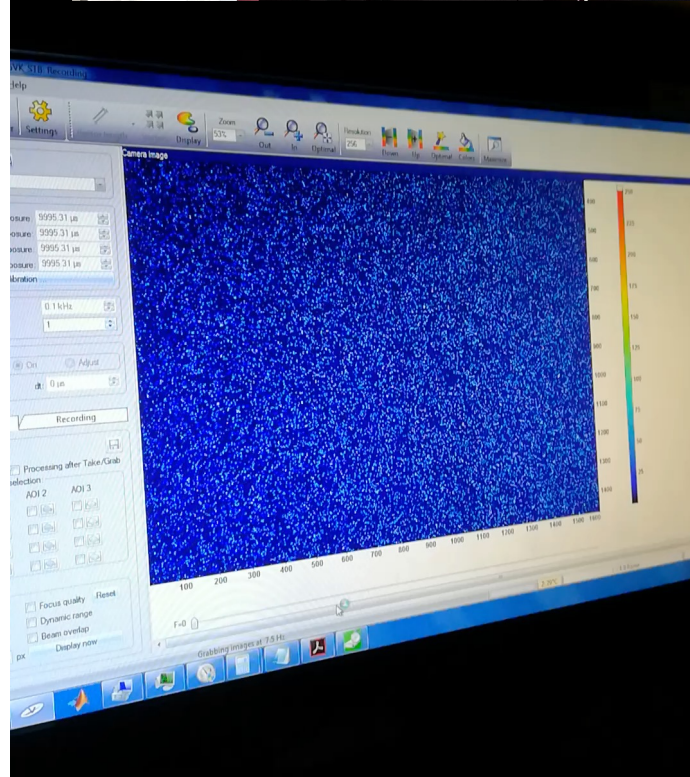
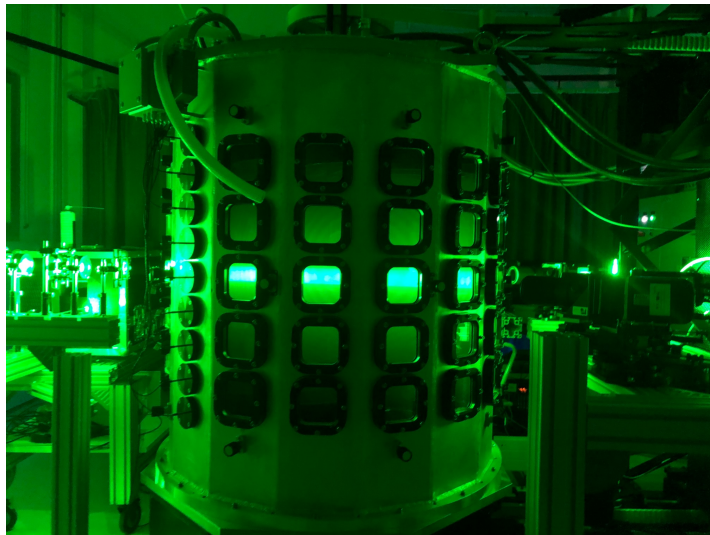
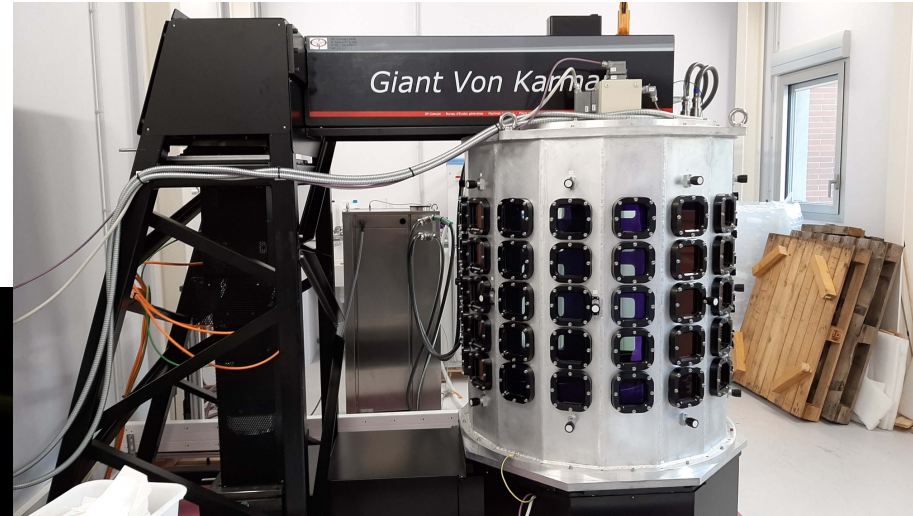
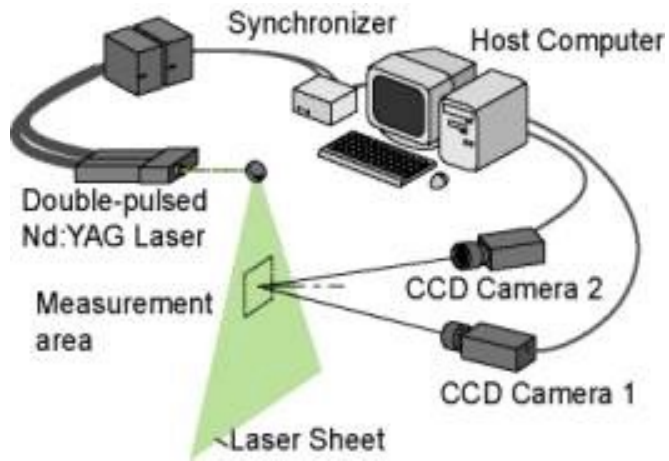
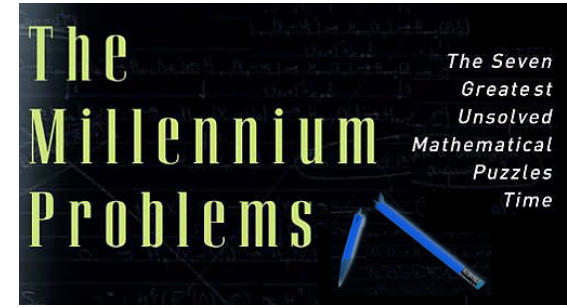


$$D(u) = (\Delta u)^3 / 12L$$



”...in three dimensions a mechanism for complete dissipation of all kinetic energy, even without the aid of viscosity, is available.”

A la recherche des empreintes de singularités...



Saw et al. (2016), Nature-Comm. 7
Cheminet et al, PRL 2022

Quel est le problème avec la turbulence?



Navier-Stokes et Problème pratique

Equations de Navier-Stokes

Peut-on simuler tous les écoulements avec Navier-Stokes?

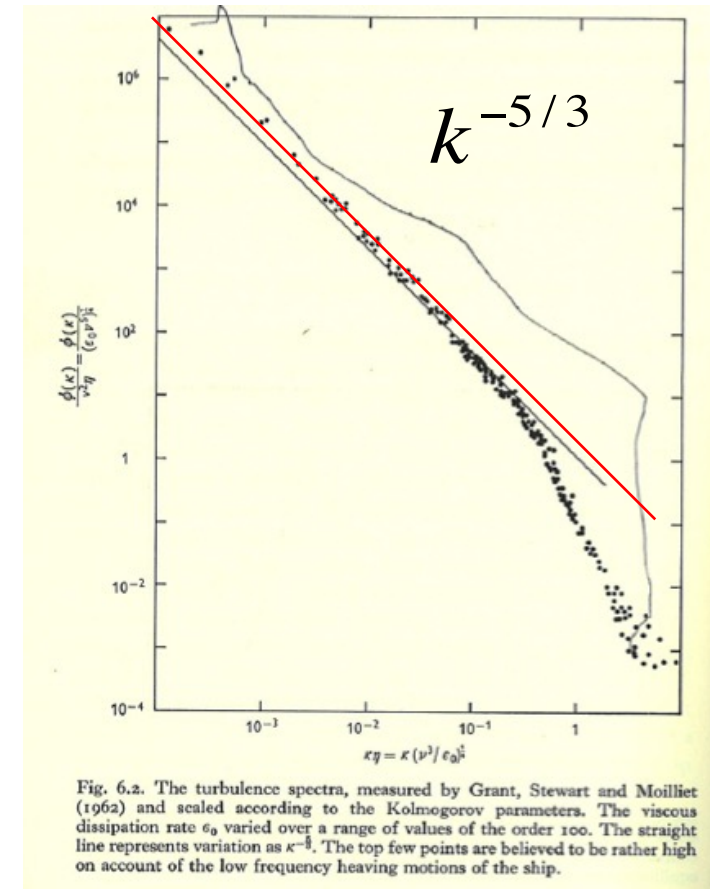
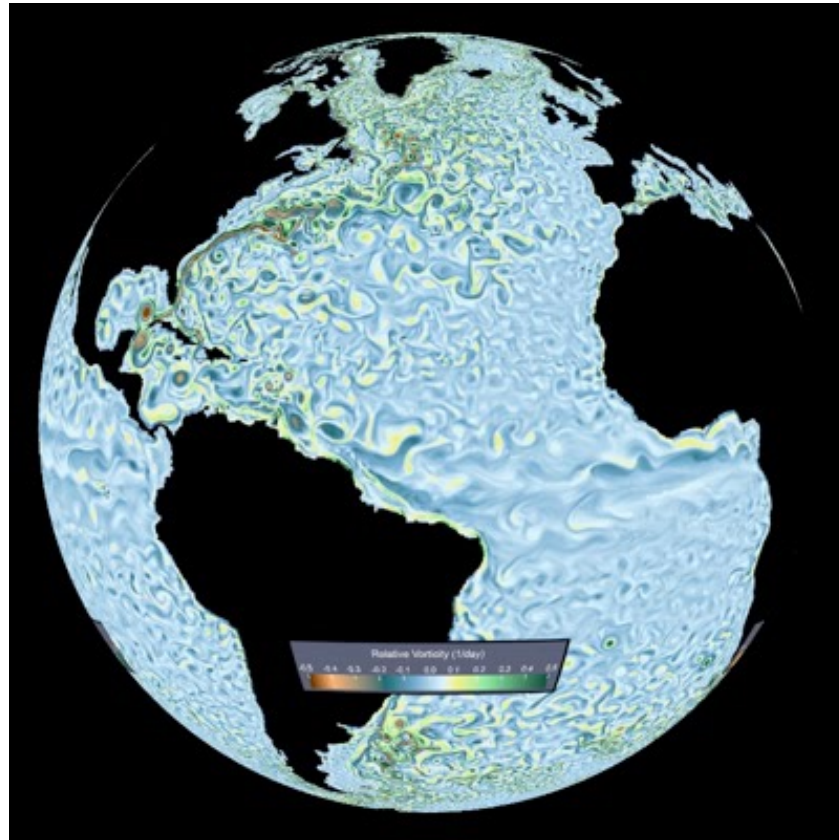
$$\vec{\nabla} \cdot \vec{u} = 0$$

$$\partial_t \vec{u} + (\vec{u} \cdot \vec{\nabla}) \vec{u} = -\frac{1}{\rho} \vec{\nabla} p + \nu \Delta \vec{u}$$

$$\text{Re} = \frac{(u \nabla u)}{\nu \Delta u} = \frac{LU}{\nu}$$

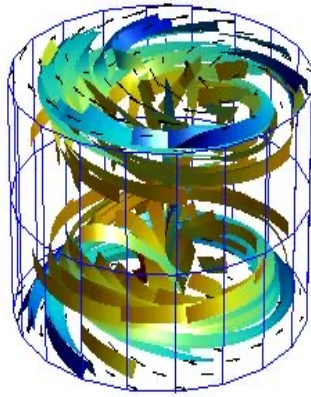
Turbulence: $\text{Re} \gg 1$

Tourbillons sur la Terre...



Spectre de Kolmogorov

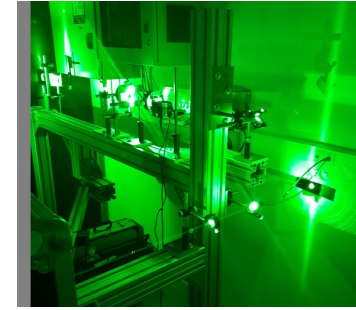
Mesures: Tourbillons dans le laboratoire



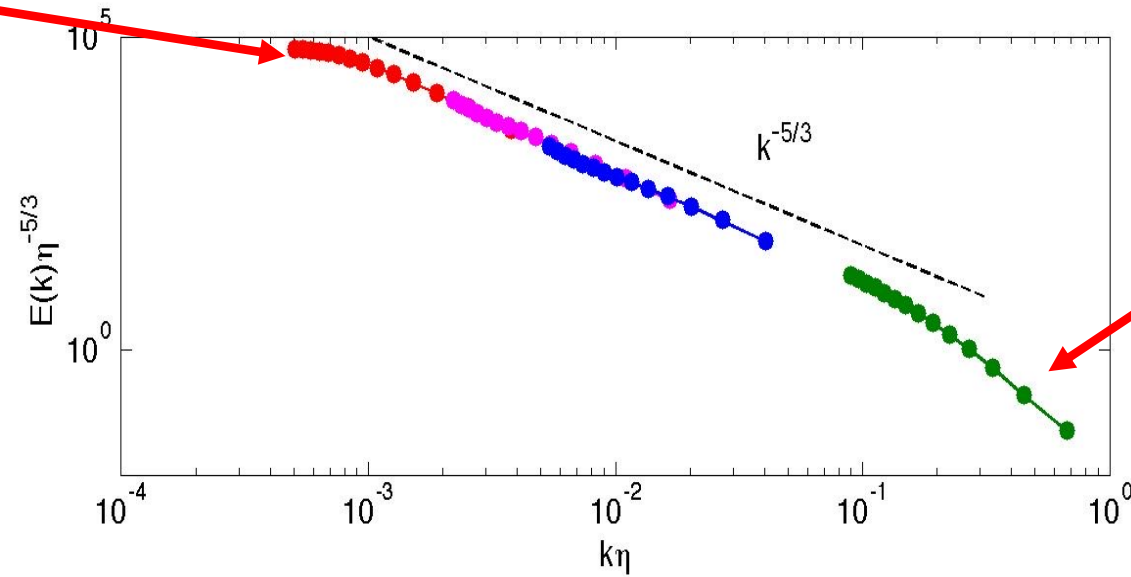
4 Cameras rapides



Laser



L=Taille du forçage

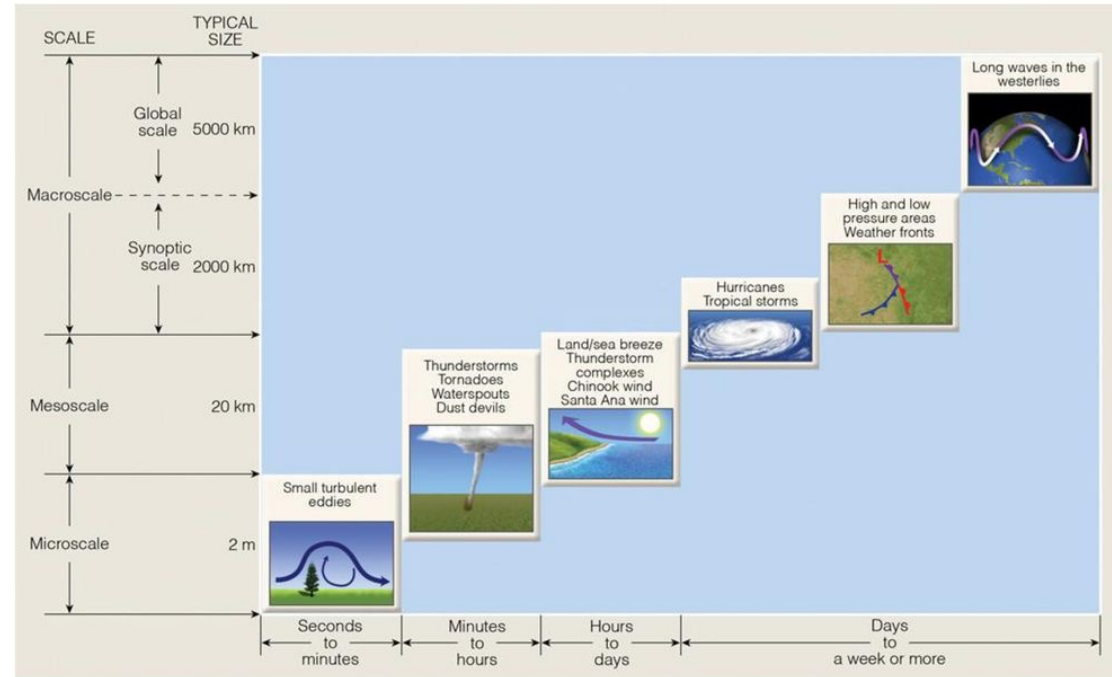
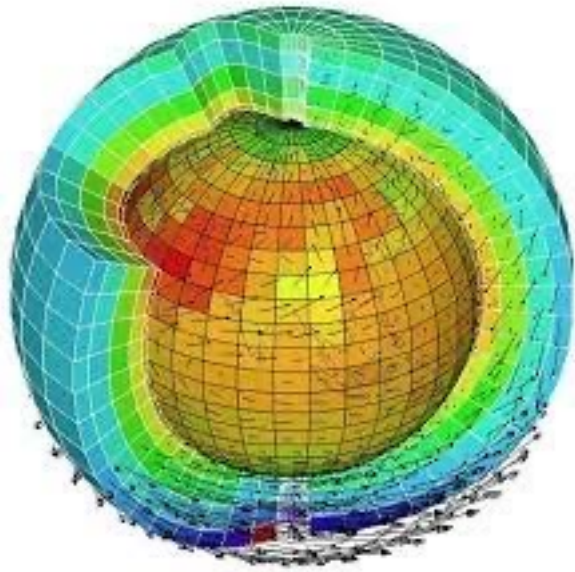


$$\eta \sim \nu^{3/4}$$

Echelle de Kolmogorov

Le problème pratique en turbulence

Scales of Motion



© Cengage Learning®

(pp. 230-231)

Ordres de grandeurs

$$L=10^3 \text{ km}$$

$$\eta= 1 \text{ mm}$$

$$(L/\eta)^3=10^{27} \text{ /pas de temps}$$

1990: 10^8
2023: 10^{13}

Simulations

Nombre de points de maille requis $(L/\eta)^3$

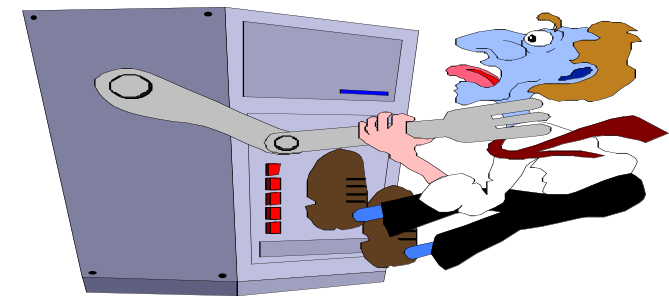
Temps de simulation et énergie

Etat de l'art pour $(L/\eta)^3=10^9$

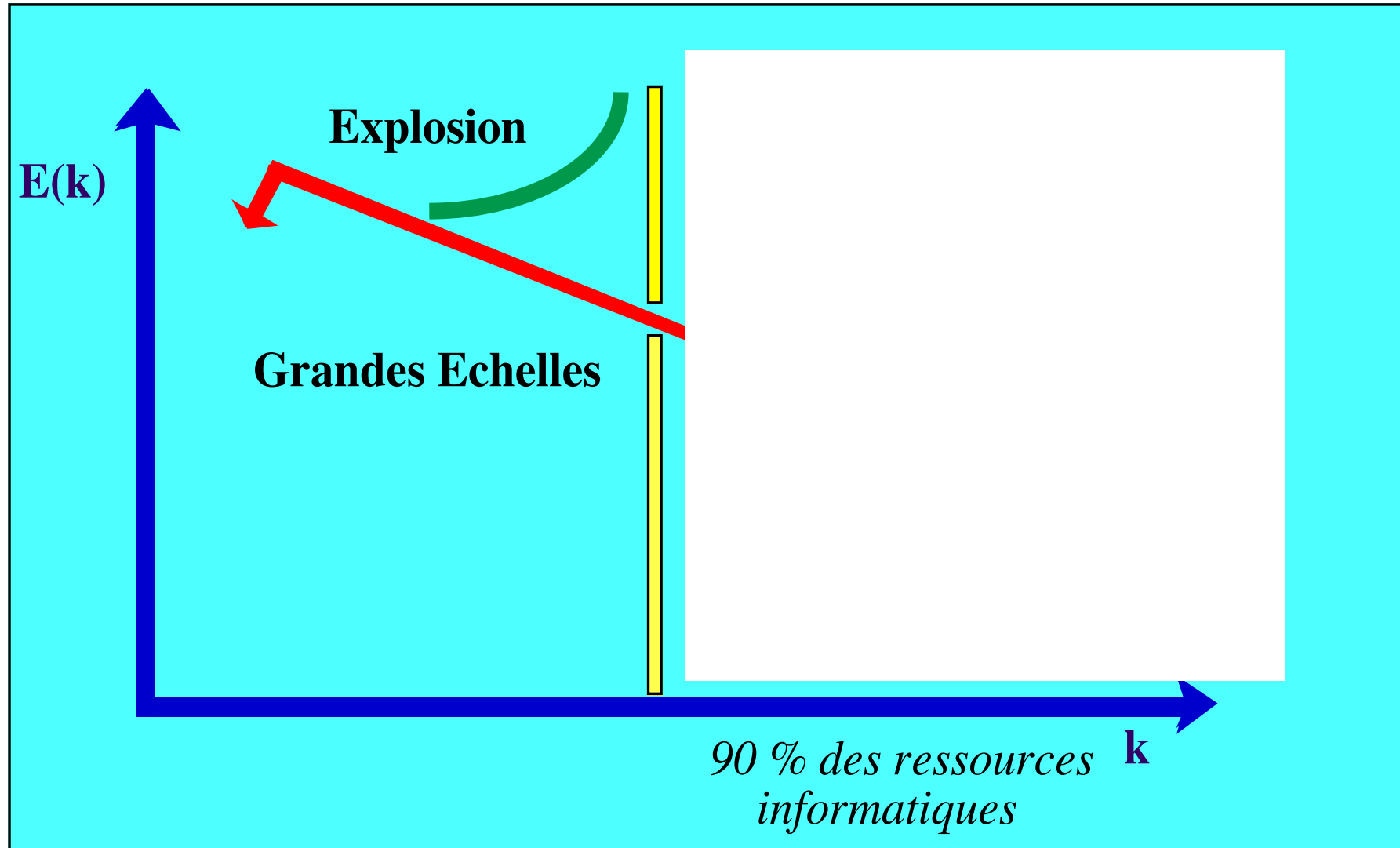
Pour 1 simu resolution 1 km , pendant 100 ans

$T_{\text{simu}}=6 \text{ ans}$; $P= 60 \text{ GWh}$ sur top super-computer avec 4888 GPU

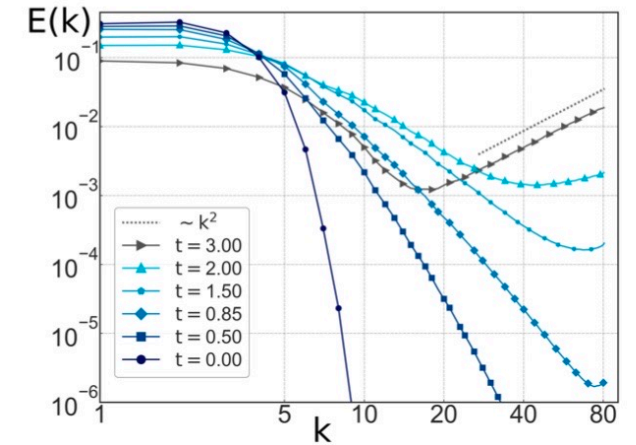
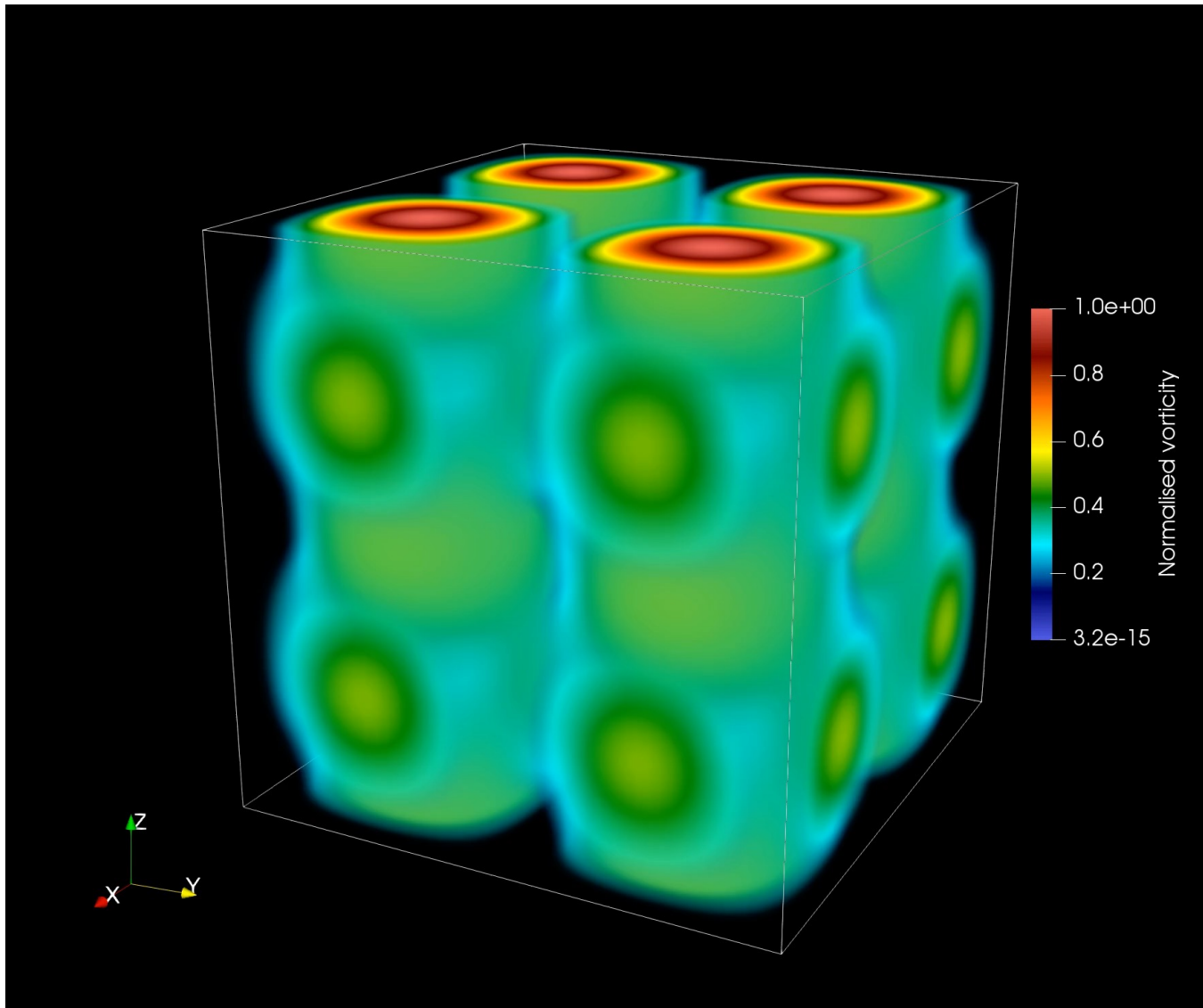
Simuler les effets induits par le changement climatique contribuerait au changement climatique!!



Que peut on faire? Tronquer?



Ce que la troncation simple provoque....

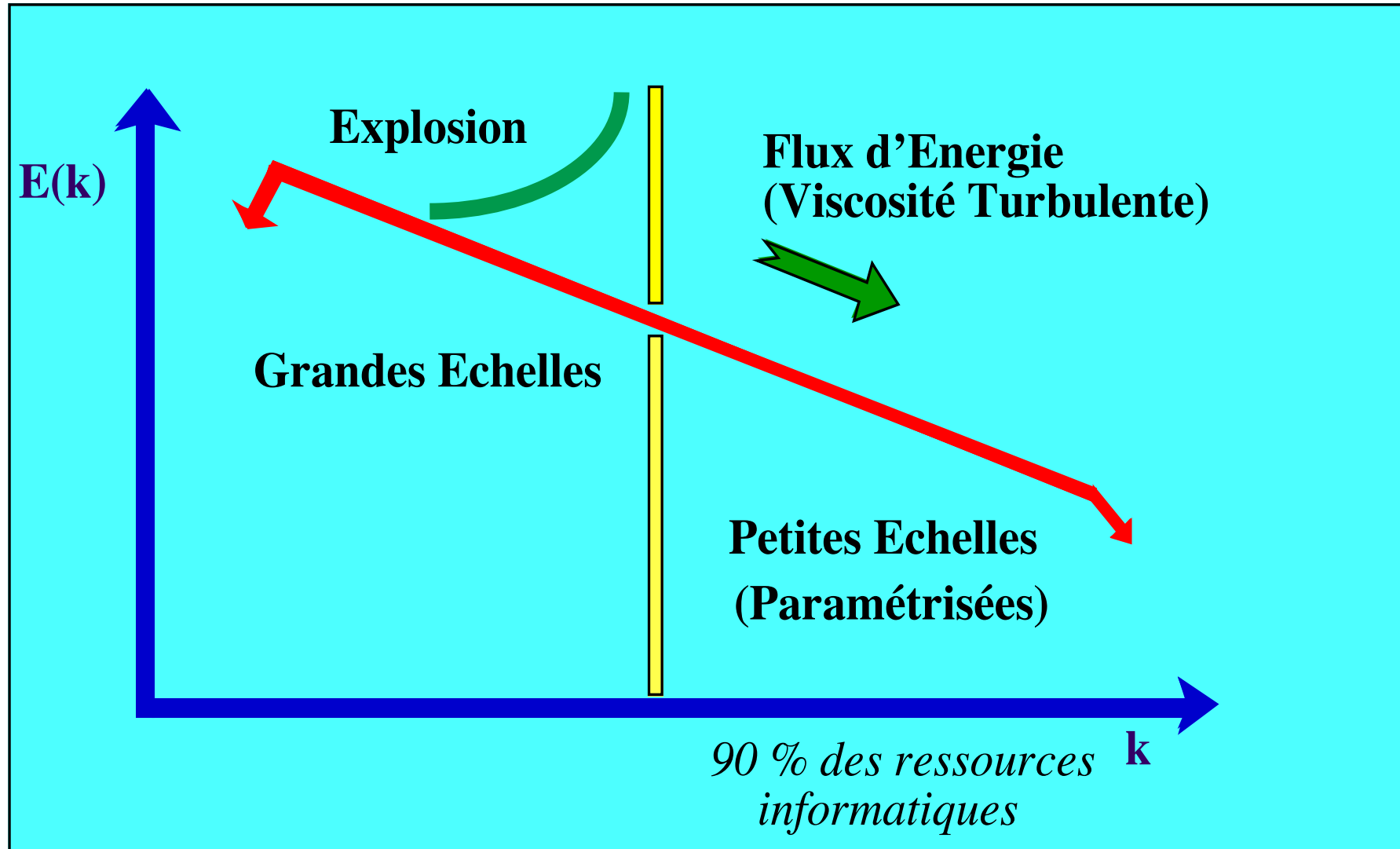


Cichowlas et al, PRL, 2005

**A cause de la troncation, toute simulation finit en
Bruit à petite échelle**

Courtesy J.I. Polanco

Que peut on faire? Paramétrer!



Exemple de paramétrisation: viscosité turbulente

Analyse dimensionnelle

$$\nu_T = K V L$$

Echelle caractéristique

Constante

Vitesse caractéristique

Si constant

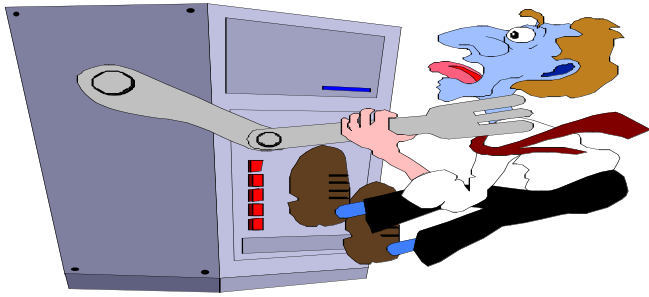


Athmosphère: x6-> goudron
Océan x 5-> miel

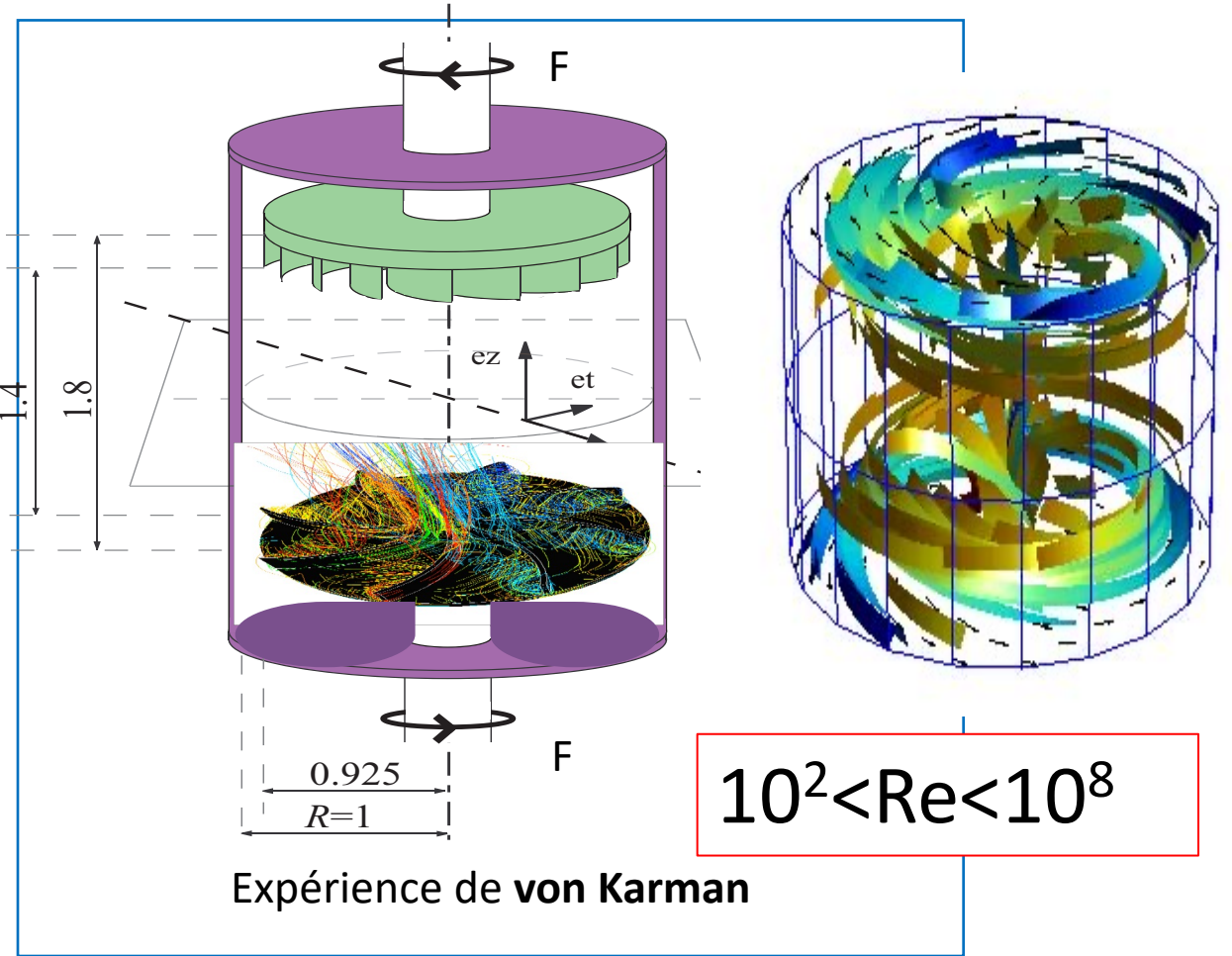
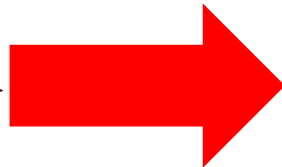


$$\vec{\nabla} \cdot \vec{u} = 0$$

$$\partial_t \vec{u} + (\vec{u} \cdot \vec{\nabla}) \vec{u} = -\frac{1}{\rho} \vec{\nabla} p + \nu \Delta \vec{u}$$



$Re < 10^5$



$10^2 < Re < 10^8$

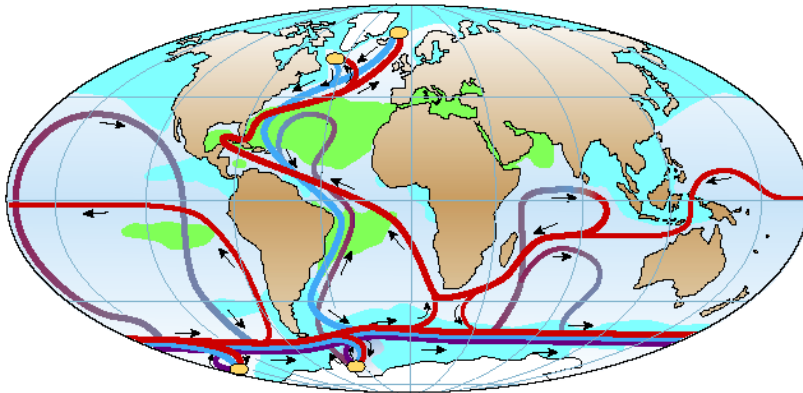
On a (encore) besoin d'expériences pour comprendre la turbulence!

Résultat VKE

Collaboration SPEC/LSCE

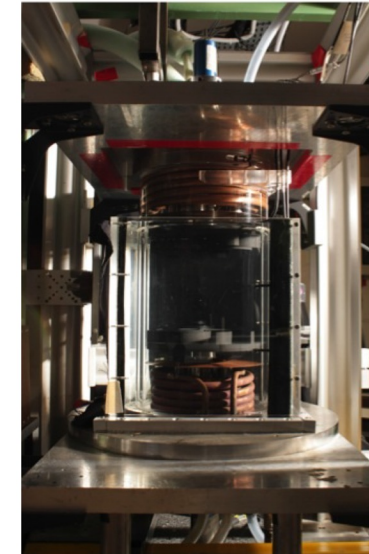
Saint-Michel et al, PRL, 2013
Faranda et al, PRL, 2017

Forçage Solaire
Transport de Chaleur
Circulation Océanique



Mesure de Température

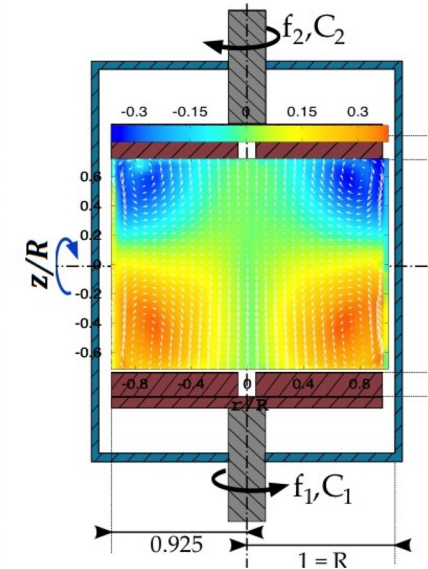
Circulation océanique



Mesure de vitesse

Circulation turbulente

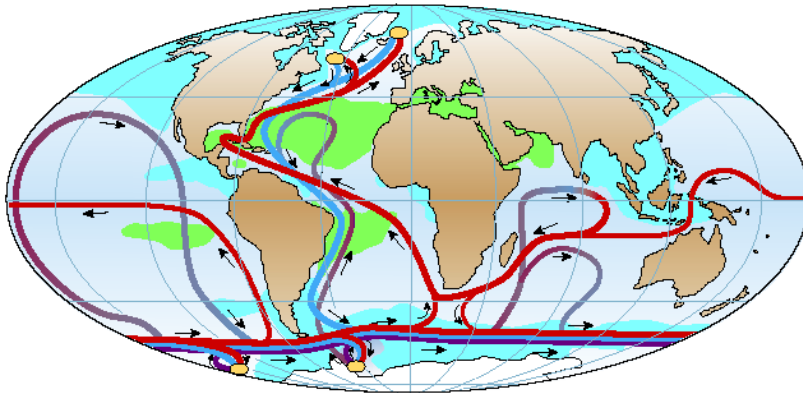
Forçage par Turbine
Transport Mom. Ciné
Circulation moyenne



Résultat VKE

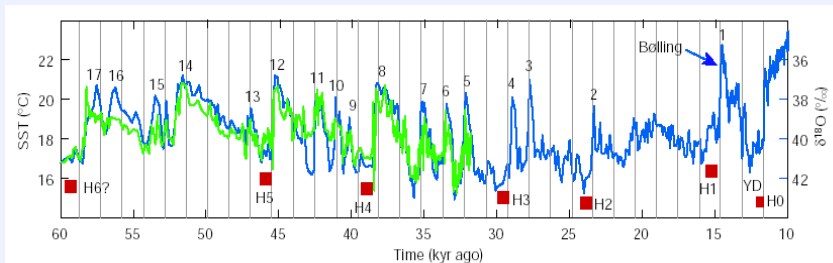
Collaboration SPEC/LSCE

Forçage Solaire
Transport de Chaleur
Circulation Océanique



Mesure de Température

Figure 3 Temperature reconstructions from ocean sediments and Greenland ice. Proxy data from the subtropical Atlantic⁸⁶ (green) and from the Greenland ice core GISP2 (ref. 87; blue) show several Dansgaard-Oeschger (D/O) warm events (numbered). The timing of Heinrich events is marked in red.



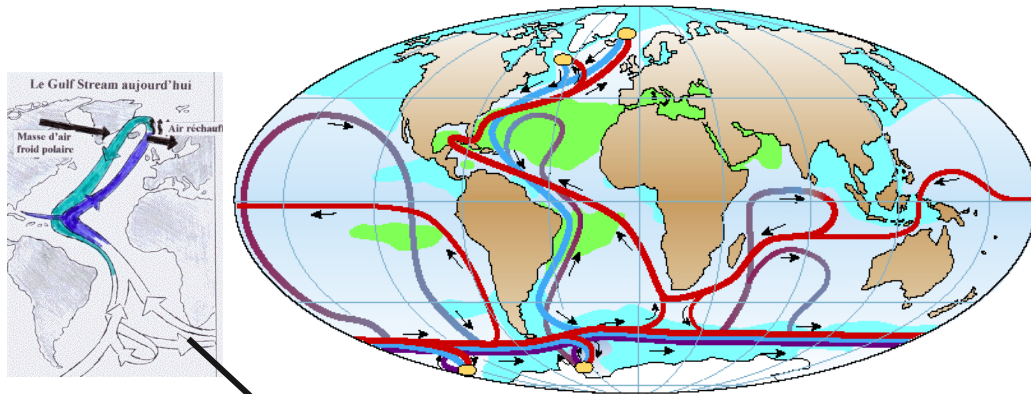
Grey lines at intervals of 1.470 years illustrate the tendency of D/O events to occur with this spacing, or multiples thereof.

Résultat VKE

Collaboration SPEC/LSCE

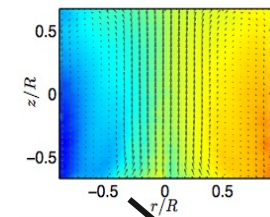
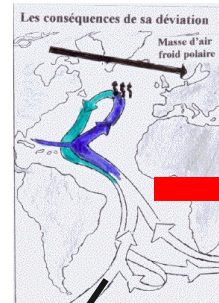
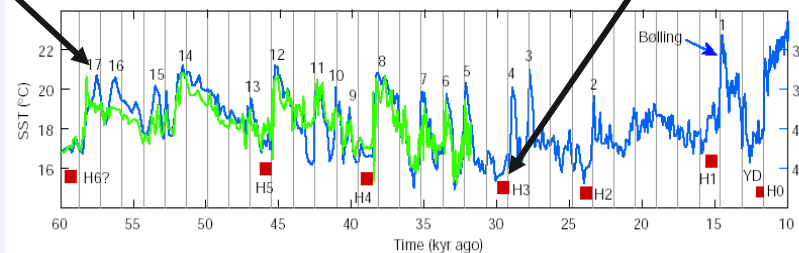
Forçage Solaire
Transport de Chaleur
Circulation Océanique

Forçage par Turbine
Transport Mom. Ciné
Circulation moyenne

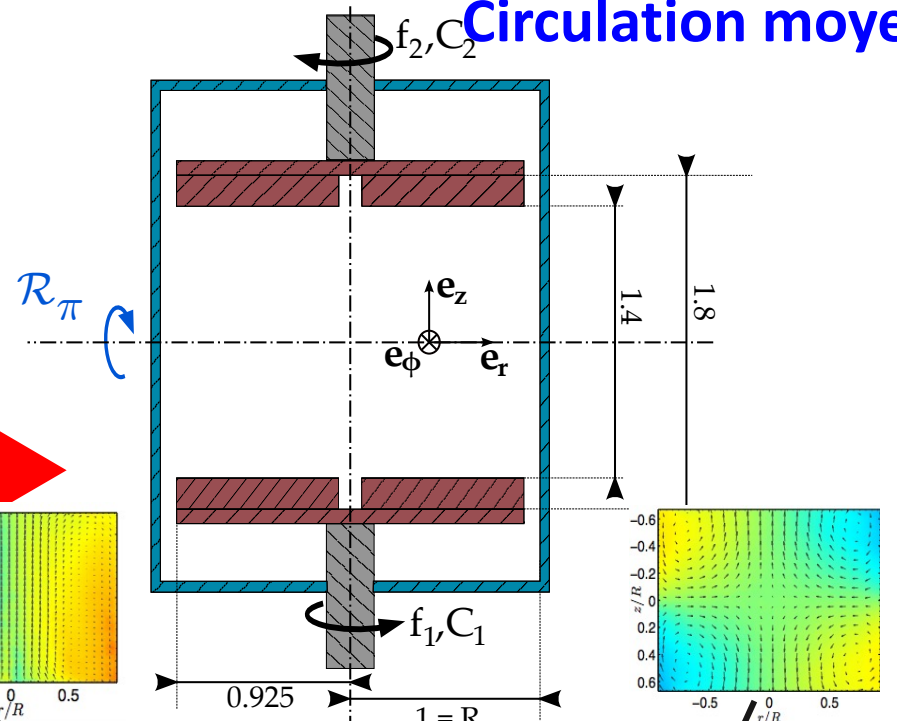
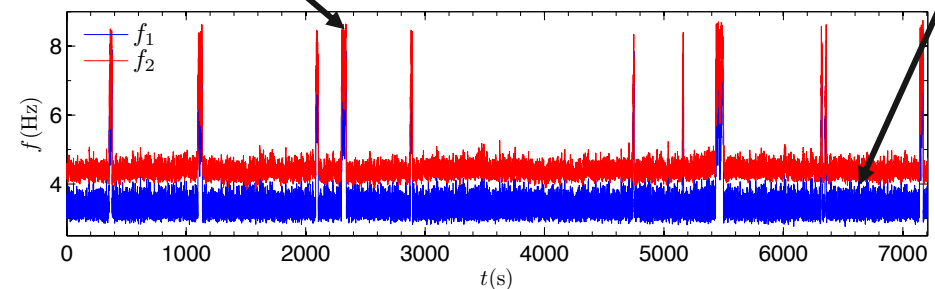


Mesure de Température

Figure 3 Temperature reconstructions from ocean sediments and Greenland ice. Proxy data from the subtropical Atlantic⁸⁶ (green) and from the Greenland ice core GISP2 (ref. 87; blue) show several Dansgaard-Oeschger (D/O) warm events (numbered). The timing of Heinrich events is marked in red. Grey lines at intervals of 1.470 years illustrate the tendency of D/O events to occur with this spacing, or multiples thereof.



Mesure de vitesse



Conclusion

On a les équations mais ça ne nous aide pas!-> expériences de laboratoire
La paramétrisation est difficile-> travail théorique!

La turbulence restera encore mystérieuse pour longtemps!